OPERACIONES CON MATRICES FICHA 1

1) Calcula la matriz X tal que $\begin{pmatrix} 2 & -1 \\ -1 & -1 \end{pmatrix} + 3X = \frac{1}{2}\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$

2) Comprueba que la matriz $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 3 \\ 2 & 1 & 1 \end{pmatrix}$ cumple $-A^3 + 6A^2 - 8A + 6I = 0$

3) Calcula las matrices X e Y, solución del sistema:

$$\left.\begin{aligned} 2X - 3Y &= \begin{pmatrix} 5 & 1 \\ 11 & -4 \end{pmatrix} \\ X + Y &= \begin{pmatrix} \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \end{aligned}\right\}$$

4) Halla a y b para que se cumpla: $\begin{pmatrix} 1 & 2 \\ a & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} b \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

5) Calcula todas las matrices X que conmuten con $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

SOLUCIONES

1) $\begin{pmatrix} 2 & -1 \\ -1 & -1 \end{pmatrix} + 3\mathrm{X} = \frac{1}{2}\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$

$$\rightarrow 3X = \frac{1}{2}\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 1 \\ 1 & -\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -\frac{5}{2} & 2 \\ 2 & \frac{1}{2} \end{pmatrix} \Rightarrow$$

$$\rightarrow X = \frac{1}{3}\begin{pmatrix} -\frac{5}{2} & 2 \\ 2 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{5}{6} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{6} \end{pmatrix}$$

2) $$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 3 \\ 2 & 1 & 1 \end{pmatrix} \rightarrow A^2 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 3 \\ 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 3 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 3 \\ 6 & 12 & 12 \\ 6 & 6 & 4 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 4 & 5 & 3 \\ 6 & 12 & 12 \\ 6 & 6 & 4 \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 3 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 14 & 22 & 18 \\ 36 & 54 & 48 \\ 20 & 28 & 22 \end{pmatrix}$$

$$-A^3 + 6A^2 - 8A + 6I = -\begin{pmatrix} 14 & 22 & 18 \\ 36 & 54 & 48 \\ 20 & 28 & 22 \end{pmatrix} + 6\begin{pmatrix} 4 & 5 & 3 \\ 6 & 12 & 12 \\ 6 & 6 & 4 \end{pmatrix} - 8\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 3 \\ 2 & 1 & 1 \end{pmatrix} + 6I =$$

$$= \begin{pmatrix} -14 & -22 & -18 \\ -36 & -54 & -48 \\ -20 & -28 & -22 \end{pmatrix} + \begin{pmatrix} 24 & 30 & 18 \\ 36 & 72 & 72 \\ 36 & 36 & 24 \end{pmatrix} + \begin{pmatrix} -16 & -8 & 0 \\ 0 & -24 & -24 \\ -16 & -8 & -8 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \rightarrow$$

$-A^3 + 6A^2 - 8A + 6I = 0$

3) $$\left.\begin{aligned} 2\mathrm{X} - 3\mathrm{Y} &= \begin{pmatrix} 5 & 1 \\ 11 & -4 \end{pmatrix} \\ \mathrm{X} + \mathrm{Y} &= \begin{pmatrix} \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \end{aligned}\right\} \begin{aligned} &\rightarrow \left.\begin{aligned} 2X - 3Y &= A \\ X + Y &= B \end{aligned}\right\} \rightarrow \left.\begin{aligned} 2X - 3Y &= A \\ 3X + 3Y &= 3B \end{aligned}\right\} \rightarrow 5X = A + 3B \\ &Y = B - X \Rightarrow Y = B - \frac{1}{5}(A + 3B) \end{aligned}$$

$$X = \frac{1}{5}(A + 3B) = \frac{1}{5}\begin{pmatrix} 5 & 1 \\ 11 & -4 \end{pmatrix} + \frac{3}{5}\begin{pmatrix} \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{5} \\ \frac{11}{5} & -\frac{4}{5} \end{pmatrix} + \begin{pmatrix} \frac{1}{5} & -\frac{3}{10} \\ \frac{3}{10} & -\frac{3}{10} \end{pmatrix} = \begin{pmatrix} \frac{6}{5} & -\frac{1}{10} \\ \frac{19}{10} & -\frac{11}{10} \end{pmatrix}$$

$$Y = B - X = \begin{pmatrix} \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} - \begin{pmatrix} \frac{6}{5} & -\frac{1}{10} \\ \frac{19}{10} & -\frac{11}{10} \end{pmatrix} = \begin{pmatrix} -\frac{13}{15} & -\frac{2}{5} \\ -\frac{7}{5} & \frac{3}{5} \end{pmatrix}$$

4) $\begin{pmatrix} 1 & 2 \\ \text{a} & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \text{b} \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \to \begin{pmatrix} 5 \\ a+6 \end{pmatrix} + \begin{pmatrix} b \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \to \left.\begin{matrix} 5+b=5 \\ a+6-3=2 \end{matrix}\right\} \to \left.\begin{matrix} b=0 \\ a=-1 \end{matrix}\right\}$

$$AX = XA \to \left\{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix}\right\} = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right\} \to$$

5) $\left\{\begin{matrix} a-2c = a+3b \to & -3b+2c = 0 & \\ b+2d = 2a+4b \to & -2a-3b+2d = 0 & \\ 3a+4c = c+3d \to & 3\text{a} + 3\text{c} - 3\text{d} = 0 \leftarrow & \text{simplificando} \\ 3b+4d = 2c+4d \to & 3\text{b} - 2\text{c} = 0 \leftarrow & \text{es proporcional a } \text{E}_1 \end{matrix}\right\}$

Tenemos que encontrar una matriz X tal que $AX = XA$, es decir, tenemos que encontrar los números a, b, c y d tales que $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, de donde:

$$\left.\begin{matrix} a-2c = a+3b \\ b+2d = 2a+4b \\ 3a+4c = c+3d \\ 3b+4d = 2c+4d \end{matrix}\right\} \to \left.\begin{matrix} -3b+2c = 0 \\ -2a-3b+2d = 0 \\ 3a+3c-3d = 0 \\ 3b-2c = 0 (= E_1) \end{matrix}\right\} \xrightarrow{reordenando} \left.\begin{matrix} a+c-d = 0 \\ -2a-3b+2d = 0 \\ 3b-2c = 0 \end{matrix}\right\} \xrightarrow{Gauss}$$

$$\begin{pmatrix} 1 & 0 & 1 & -1 & 0 \\ -2 & -3 & 0 & 2 & 0 \\ 0 & 3 & -2 & 0 & 0 \end{pmatrix} \xrightarrow{F_2+2F_1} \begin{pmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & -3 & 2 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \end{pmatrix} \xrightarrow{F_3+F_2} \begin{pmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & -3 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\left.\begin{matrix} a+c-d = 0 \\ -3b+2c = 0 \end{matrix}\right\} \to \left.\begin{matrix} a = t \\ c = s \end{matrix}\right\} \Rightarrow \left.\begin{matrix} b = \frac{2}{3}s \\ d = t+s \end{matrix}\right\} \to X = \begin{pmatrix} t & \frac{2}{3}s \\ s & t+s \end{pmatrix} \text{forma general}$$